EX15

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

1) $Df = IR$

1) $\forall x \in Df$ on a $(-x) \in Df$

$$f(-x) = \frac{e^{-x} - e^x}{e^{-x} + e^x} = -f(x)$$

Donc f est impaire

2) $f'(x) = \frac{4}{(e^x + e^{-x})^2} > 0$

x	$-\infty$		$+\infty$
$f_n'(x)$		$+$	
$f_n(x)$	-1	↗	1

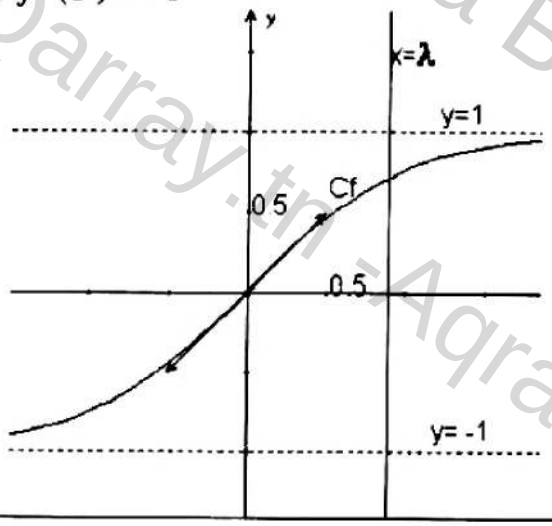
$\lim_{x \to +\infty} f(x) =$

$$= \lim_{x \to +\infty} \left(\frac{1 - \frac{1}{e^{2x}}}{1 + \frac{1}{e^{2x}}} \right) = 1$$

($1 - \frac{1}{e^{2x}} \to 0$, $1 + \frac{1}{e^{2x}} \to 0$)

$\lim_{x \to -\infty} f(x) = -1$ car f est impaire

b) $f'(0) = 1$



3) $\lambda > 1$, $\Delta : x = \lambda$

$$A(\lambda) = \int_0^{\lambda} f(x).dx$$

$$= \left[Ln(e^x + e^{-x}) \right]_0^{\lambda}$$

$$= Ln(e^{\lambda} + e^{-\lambda}) - Ln\, 2$$

$$A(\lambda) = Ln\left(\frac{e^{\lambda} + e^{-\lambda}}{2} \right) \text{ u.a}$$

EX16

1) a/ on pose $u(x) = e^x - x - 1$; $x \in IR$

$u'(x) = e^x - 1 > 0 \Leftrightarrow x > 0$

x	$-\infty$		0		$+\infty$
$u'(x)$		$-$	0	$+$	
$u(x)$	$+\infty$	↘	0	↗	$+\infty$

u admet 0 comme minimum absolu en 0

$\Rightarrow u(x) \geq 0$; $\forall x \in IR$

$\Rightarrow u(x) > 0$; $\forall x \in IR^*$

$\Rightarrow e^x - x - 1 > 0 \Rightarrow e^x > x + 1$

b) $x \neq 0$

(i) $t = -x$ d'après a) $e^t > t + 1$

$\Rightarrow e^{-x} > 1 - x$

(ii) pour $x \in]0, 1[$

$e^{-x} > 1 - x \Rightarrow \frac{1}{e^x} > 1 - x$ comme $1 - x > 0$

on aura : $e^x < \frac{1}{1 - x}$

2) $n \in IN^*$, $U_n = (1 + \frac{1}{n})^n$

$V_n = (1 + \frac{1}{n})^{n+1}$

Qarray.tn - Aqra Blech- Qarray.tn